Class notes on Aircraft Propeller

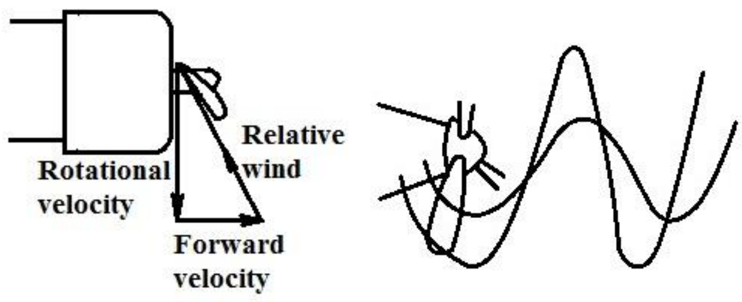
2.7 Propeller theory: Introduction

The success of today's high performance low-speed airplanes lies in the invention and development of modern controllable pith propeller. The aircraft propeller is essentially a series of rotating wings or airfoils of equal length, which meets at a common central hub attached to the crankshaft of the aircraft engine. Whereas the airfoil section of an aircraft wing is designed to produce lift efficiently, the propeller section is designed to produce thrust to drive an aircraft forward.

The resultant of forward aircraft velocity and the rotational velocity of the propeller is called the *relative wind* and this is actually faced by the propeller blades. A propeller blade actually traverses a spiral path because it is having a simultaneous forward and rotational motion, much like the motion of a screw. Also a point near the tip of a blade will trace a larger distance than a point near the shank of the blade. If these path is to be traced by the blade elements at their most efficient angle (i.e. at highest aerodynamic efficiency), the blade element angles need to be changed, larger towards the shank and smaller towards the tip of the blade. The gradual change in blade element angles is known as the *pitch distribution*. The relative wind and spiral path traversed by a blade element are shown in figure 2.6.

The power requirement of an airplane varies during the course of its flight. Larger engine power creates larger rotational speed of the propeller and consequently larger lift and drag on the blades. Large variations in the engine speed reduce its life. Ideally a gear shift mechanism, like a motor-bike, should have been included between the engine crankshaft and the propeller disk. Instead of that, however, the blades are allowed to rotate about their axis in a socket, which is controlled by a separate engine-operated *governor* and the blade angles are changed at different flight conditions (i.e. take-off, cruising) so that the aerodynamic efficiency is maximum, while the engine speed is kept constant. Most modern low speed airplanes are driven by *constant speed propellers*.

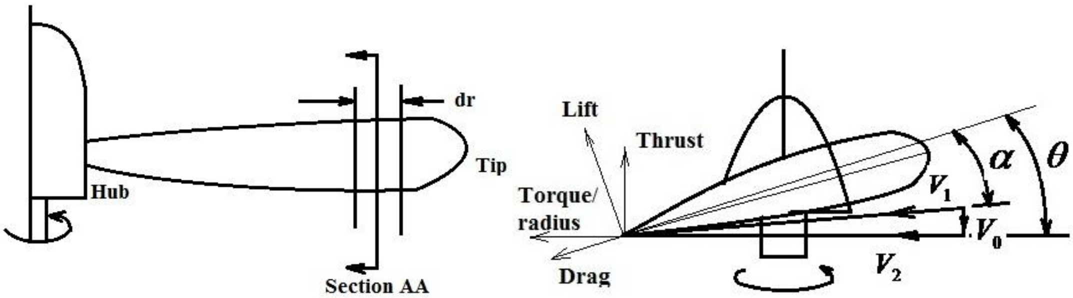
2.8 Glauert blade element theory and performance



(a) (b)

Figure 2.6 (a) Relative wind and (b) helical path traversed by a blade element.

In the blade element theory of Glauert, the propeller is divided into a number of independent sections along the length. Then a force balance is applied among lift, drag, thrust and torque per unit radius produced by each section. Balancing of axial and angular momentum is also carried out. The resulting non-linear equations are solved by iterative techniques. The section thrust and torque are then summed up to predict the overall performance of the propeller. Because of the two-dimensional treatment of the flow-field, this theory over predicts the thrust and under predicts the torque with the result of 5-10% increase in theoretical efficiency over measured performance. However, the theory is used widely for a good first order prediction of thrust, torque and efficiency for a large range of operating conditions.



(a) (b)

Figure 2.7 (a) Typical blade element (b) Force and flow vectors on an element

$V_0 \equiv$ axial flow velocity at propeller disk, $V_2 \equiv$ angular flow velocity vector, $V_1 \equiv$ section local flow velocity vector $\equiv$ summation of vectors V_0 and V_2

The lift and drag of a section are calculated using standard 2-D airfoil properties.

$$\phi = \theta - \alpha$$

$$\Delta T = \Delta L \cos\phi - \Delta D \sin\phi, \tfrac{\Delta Q}{r} = \Delta D \cos\phi + \Delta L \sin\phi$$

$$\Delta L = C_L \tfrac{1}{2}\rho V_1^2 cdr, \Delta D = C_D \tfrac{1}{2}\rho V_1^2 cdr, c \text{ is the blade chord, lift producing area is } cdr$$

Substituting the values of ΔL, ΔD and assuming the number of blades to be B, one gets

$$\Delta T = \tfrac{1}{2}\rho V_1^2 c(C_L \cos\phi - C_D \sin\phi)Bdr \text{ --------(1)}$$

$$\Delta Q = \tfrac{1}{2}\rho V_1^2 c(C_L \sin\phi - C_D \cos\phi)Brdr \text{ -------(2)}$$

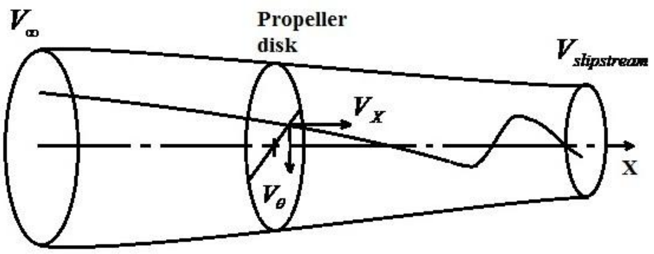


Figure 2.8 Typical streamtube passing through a propeller

$$V_0 = V_\infty + aV_\infty \text{ where } a \text{ is the axial inflow factor}$$
$$\mathrm{V}_2 = \Omega r - b\Omega r \text{ where } b \text{ is the angular inflow factor}$$

$$\mathrm{V}_1 = \sqrt{V_0^2 + V_2^2} \text{ --------(3)}$$

$$\alpha = \theta - \tan^{-1}(V_0 / V_2) \text{ --------(4)}$$

2.9 Axial and angular conservation of momentum

The stream-tube cross section used in the following calculation is the area swept out by a blade element. Also all variables are assumed to be time averaged values.

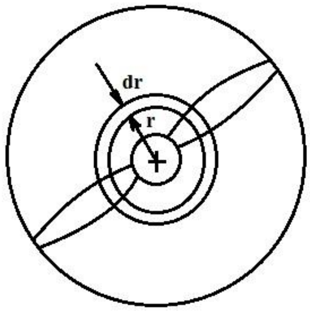


Figure 2.9 Frontal view of the area swept by propeller blades.

$$\Delta T = \text{change in momentum flow rate}$$
$$= \text{mass flow rate in the tube} \times \text{change in velocity}$$
$$= \rho 2\pi r dr V_0 (V_{slipstream} - V_\infty)$$

It can be shown that

$$V_0 = \frac{V_\infty + V_{slipstream}}{2} \text{ so that}$$
$$V_{slipstream} = V_\infty (1 + 2a)$$

So

$$\Delta T = \rho 2\pi r V_\infty (1 + a)[V_\infty (1 + 2a) - V_\infty]dr$$
$$= 4\pi r \rho \mathrm{V}_\infty^2 (1 + a)a dr \text{ --------- (5)}$$

For angular momentum

$$\Delta Q = \text{rate of change of angular momentum} \times \text{radius}$$
$$= \text{mass flow rate in the stream tube} \times \text{change in circumferential velocity} \times \text{radius}$$
$$= \rho 2\pi r \mathrm{drV}_0 [V_{\theta,slipstream} - 0_{free-stream}]r$$

It can be shown that

$$V_{\theta,slipstream} = 2b\Omega r$$

So

$$\Delta Q = \rho 2\pi r V_\infty (1+a)(2b\Omega r) r dr$$
$$= 4\pi r^3 \rho V_\infty (1+a) b\Omega dr \text{ -----------} (6)$$

2.10 Iterative solution technique

Now we have six non-linear coupled systems of equations and four unknowns, namely $\Delta Q, \Delta T, a, b$. So it is possible to go for an iterative improvement of the unknowns.

a) Assume some initial values of a and b.

b) Use these values to find flow angles on the blade using equations 3 and 4.

c) Use blade section properties to find the thrust and torque using equations 1 and 2.

d) Use equations 5 and 6 to find improved estimates of a and b.

e) Repeat the procedure until values of a and b have converged.

2.11 Propeller thrust and torque coefficients, efficiency

The overall thrust and torque of the blade can be obtained by summing up the values for radial blade element values i.e.

$$T = \sum \Delta T \text{ and } Q = \sum \Delta Q$$

The non-dimension thrust co-efficient, torque co-efficient and advance ratios are given by

$$C_T = T / \rho n^2 D^4, C_Q = Q / \rho n^2 D^5, J = V_\infty / nD$$

and

$$\eta_{propeller} = (J / 2\pi)(C_T / C_Q)$$

where n is the rotational speed of the propeller in rev/s and D is the propeller diameter.